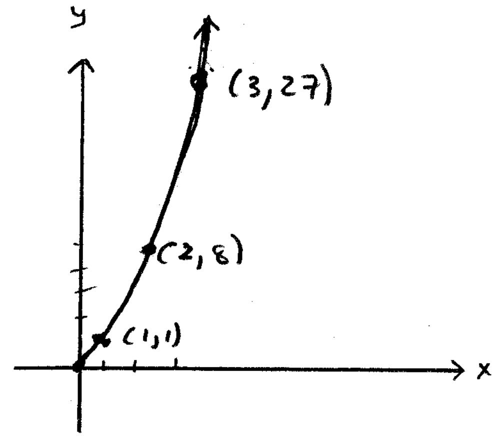
Answers
J11 pps 2-4
Probs #1-2

p. 2

[1]

x	0	1	2	3	4	5
x^3	0	1	8	27	64	125



p. 4

[2]

$$y = x^n \begin{cases} n \text{ even}, & \text{RNG} = \{y \in \mathbb{R} \ni y \geq 0\} \\ n \text{ ODD}, & \text{RNG} = \mathbb{R} \end{cases}$$

ANSWERS

J 11 pps 5–7

Probs 1–5

p. 6

[1]

(1) $2, -2$

(2) $\emptyset$

(3) 3

(4) -3

p.7

[2]

(1) $\sqrt[4]{3}\,\sqrt[4]{27} = 3^{1/4} \cdot 3^{\frac{3}{4}} = 3^{1} = 3$

-OR-

$\sqrt[4]{3}\,\sqrt[4]{3^3} = \sqrt[4]{3^4} = 3$

(2) $\sqrt[3]{200} \div \sqrt[3]{25} = \sqrt[3]{\frac{200}{25}} = \sqrt[3]{8} = 2$

(3) $\sqrt[3]{0.01}\,\sqrt[3]{0.1} = \sqrt[3]{.001} = \sqrt[3]{\frac{1}{1000}} = \sqrt[3]{\frac{1}{10^3}} = .1$

[3]

(1) $\sqrt[3]{54} = \sqrt[3]{2 \cdot 27} = 3 \cdot \sqrt[3]{2}$

(2) $\sqrt[4]{48} = \sqrt[4]{4 \cdot 16} = 2\sqrt[4]{4}$

(3) $\sqrt[3]{500} = \sqrt[3]{4 \cdot 125} = 5\sqrt[3]{4}$

$$\begin{array}{r|l} 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

$500 = 2^2 \cdot 5^3$

[4]

Prove $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$, $a>0$, $m,n \in \mathbb{Z}^+$

Proof:

Note that $\left[(\sqrt[n]{a})^m\right]^n = a^m$, because

$$\left[(\sqrt[n]{a})^m\right]^n = (\sqrt[n]{a})^{mn} = (\sqrt[n]{a})^{nm} = \left[(\sqrt[n]{a})^n\right]^m = a^m.$$

then $\text{RHS} = \sqrt[n]{a^m}$

$$= \sqrt[n]{\left[(\sqrt[n]{a})^m\right]^n}$$

$$= (\sqrt[n]{a})^m$$

$$= \text{LHS}$$

[5]

(1) $(\sqrt[4]{25})^6 = \sqrt[4]{25^6} = \sqrt[4]{(5^2)^6} = \sqrt[4]{5^{12}} = \sqrt[4]{(5^3)^4} = 5^3 = 125$

(2) $\sqrt[3]{27^4} = (\sqrt[3]{27})^4 = 3^4 = 81$

p.9

Answers

J11 pps 8-10 # 1-5

[1]
(1) $a^{\frac{1}{4}}$, (2) $a^{\frac{3}{2}}$, (3) $a^{\frac{5}{3}}$, (4) $a^{\frac{-2}{3}}$

[2] (1) $\sqrt{a}$, (2) $\sqrt[3]{a^4}$, (3) $\sqrt[3]{\frac{1}{a^5}}$, (4) $\sqrt{a^3}$

[3]
(1) $a^{\frac{5}{4}}$, (2) $a^{-\frac{1}{4}}$, (3) a, (4) a^{-1}.

p.10

[4] $a>1$. $\dfrac{a^{-\frac{1}{3}}}{a^{-1/2}} = a^{\frac{1}{6}}$.

$1 < a \Rightarrow 1 < a^{1/6}$. so, $\dfrac{a^{-1/3}}{a^{-1/2}} > 1 \Rightarrow a^{-1/3} > a^{-1/2}$.

[5] $\dfrac{a^2}{a^{3/2}} = a^{1/2} = \sqrt{a}$.

Since $a < 1$, $\sqrt{a} < \sqrt{1}$, $\sqrt{a} < 1$.

Then $\dfrac{a^2}{a^{3/2}} < 1 \Rightarrow a^2 < a^{3/2}$. $\square$

P12

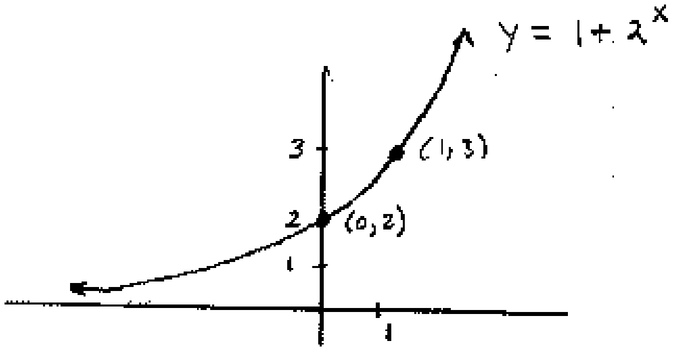
[3] $y = 2^x$ and $y = (\frac{1}{2})^x$ are symmetric w.r.t y-axis.

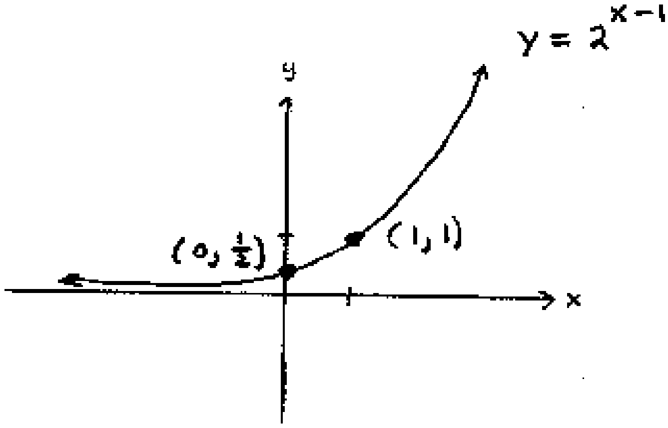
$y = 3^x$ and $y = (\frac{1}{3})^x$ " "

[4] 1 − 5, 2 − 3

P13

[5]





As always $a + f(x)$ shifts $f(x)$ up/down.

$f(x-a)$ shifts $f(x)$ left/right.

Alternatively, you know the shape, the points $(1, f(1))$, $(0, f(0))$ will fix the position of the curve.

[Solutions J11 BA]

p. 14 Exercises

[1.1] 2

[1.2] $\frac{1}{2}$

[1.3] 125

[1.4] 256

[2.1] a

[2.2] $\sqrt[6]{a}$

[2.3] $\sqrt{a}$

[3.1] $a^{-\frac{5}{6}}$

[3.2] $x^{\frac{1}{2}}$

[3.3] $\frac{y^2 + 2\,y + 1}{y}$

[6] $\sqrt{2} > \sqrt[9]{16} > \sqrt[5]{4} > \sqrt[8]{8} > \sqrt[3]{2}$

PPS 15-

P16

[1.1] $\log_5 125 = 3$ [1.2] $\log_2 .25 = -2$

[1.3] $\log_8 0.5 = -\frac{1}{3}$

[2.1] $4^{3/2} = 8$ [2.2] $64^{-1/2} = \frac{1}{8}$ [2.3] $10^0 = 1$

[3.1] 3 [3.2] $\frac{1}{2}$ [3.3] 0 [3.4] 1 [3.5] $-\frac{2}{3}$

[3.6] -2

P17 Prove $\log_a \frac{M}{N} = \log_a M - \log_a N$

Proof

Let $\log_a M = x$, $\log_a N = y$, then

$M = a^x$ and $N = a^y$. $\frac{M}{N} = \frac{a^x}{a^y} = a^{x-y}$.

Then $\log_a \frac{M}{N} = \log_a a^{x-y} = x-y \log_a a = x-y$.

$\therefore \log_a \frac{M}{N} = \log_a M - \log_a N$

$\square$

Prove $\log_a M^r = r \log_a M$

Proof

Let $x = \log_a M$,

Then $a^x = M$. So $(a^x)^r = M^r$.

$a^{rx} = M^r \equiv \log_a M^r = rx$.

Back substitution yields

$$\log_a M^r = r \log_a M$$

$\square$

P 17, ctd

[2.1] Prove $\log_a \frac{1}{N} = -\log_a N$.

Proof

Let $\log_a \frac{1}{N} = x \equiv \frac{1}{N} = a^{-x} \equiv N = a^x$.

$\log_a \frac{1}{N} = \log_a a^{-x} = -\log a^x = -\log \frac{1}{N}$.

□

[2.2] Prove $\log_a \sqrt[n]{M} = \frac{1}{n} \log_a M$

Proof

$\log_a \sqrt[n]{M} = \log_a M^{\frac{1}{n}} = \frac{1}{n} \log_a M$. □

P18

[3.1] $\log_6 \frac{9}{2} + \log_6 8 = \log_6 \left[\frac{9}{2} \cdot 8\right] = \log_6 36 = 6$

[3.2] $\log_5 250 - \log_5 2 = \log_5 \left[\frac{250}{2}\right] = \log_5 125 = 3$

[3.3] $\frac{1}{2} \log_2 25 - \log_2 10 = \log_2 25^{1/2} - \log_2 10$

$= \log_2 \frac{5}{10}$

$= \log_2 \frac{1}{2}$

$= -1$

[3.4] $\dfrac{\log_3 32}{\log_3 8} = \dfrac{\log_3 2^5}{\log_3 2^3} = \dfrac{5 \log_3}{3 \log_3} = \dfrac{5}{3}$

P18, ctd

$\log_{10} 2 = a$, $\log_{10} 3 = b$

$\log a = \log_{10} a$

NOTATION

[4.1] $\log_{10} 6 = \log(2\cdot 3) = \log 2 + \log_3 = a + b$

[4.2] $\log 5 = \log \frac{30}{6} = \log 30 - \log 6$

$= \log(3\cdot 10) - \log 6$

$= \log 10 + \log 3 - \log 6$

$= 1 + b - a - b$

$= 1 - a$

[4.3] $\log 18 = \log(2\cdot 9) = \log 2 + 3 \log 3$

$= a + 3b$

[4.4] $\log 120 = \log[10 \cdot 12] = \log 10 + \log(3\cdot 4)$

$= \log 10 + \log 3 + 2 \log 2$

$= 1 + 2a + b$

[4.5]

$\log \sqrt{.2} = \log\left[\frac{2}{10}\right]^{1/2} = \frac{1}{2}\left[\log 2 - \log 10\right]$

$= \frac{1}{2} a - \frac{1}{2}$

[5.1] Prove $\log_a b = \dfrac{1}{\log_b a}$, $a \neq 0$

Proof

$a \neq 0$

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a} \quad \square$$

[5.2] Prove $\log_a b \log_b c \log_c a = 1$, a, b, c NOT zero

Proof

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$$

$$\log_c a = \frac{\log_b a}{\log_b c}$$

so $$\log_a b \log_b c \log_c a = \frac{1}{\cancel{\log_b a}} \cdot \cancel{\log_b c} \cdot \frac{\cancel{\log_b a}}{\cancel{\log_b c}} = 1 \quad \square$$

p 19 ctd

[6.1] $$\log_2 3 = \frac{\log 3}{\log 2} = \frac{.4771}{.3011} = \boxed{1.5850}$$

[6.2] $$\log_2 10 = \frac{\log 10}{\log 2} = \frac{1}{.3011} = \boxed{3.3212}$$

[6.3] $$\log_3 20 = \log_3 (2 \cdot 10) = \log_3 2 + \log 10$$

$$= 1 + \log_3 2$$

$$\log_3 2 = \frac{\log 2}{\log 3}$$

so $$1 + \frac{.3011}{.4771} = \boxed{2.5845}$$